Acceleration of particles by rotating acceleration horizons

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We consider collision of two particles in the vicinity of the extremal acceleration horizon that includes the extremal Ker throat. It is shown that the energy in the centre of mass frame can become indefinitely large if parameters of one of particles are fine-tuned (so-called critical particle). Thus the Bañados-Silk-West (BSW) effect is possible in such space-times contrary to some recent claims in literature. As the vicinity of the extremal black hole horizon can be approximated by the metric of the acceleration one, this also gives the bridge between two kinds of the BSW effect.

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I. INTRODUCTION

The Bam̃ados-Silk-West effect (denoted hereafter as the BSW effect) discovered in 2009 [1], still attracts much attention. It consists in the possibility of getting indefinitely large energy $E_{c.m.}$ in the centre of mass frame of two particles colliding near the black hole horizon. This happens if the parameters of one of particles are fine-tuned. As $E_{c.m.}$ can be made as large as one likes, this leads to the possibility of creation of high-energetic and/or massive particles and opening new channels of reaction forbidden in laboratory conditions. Since it can give rise to unexpected radical consequences, this effect was critically examined from different viewpoints. Here, one should distinct clearly between two different issues: (i) difficulties for observations in practical astrophysics, (ii) the existence or non-existence of the effect as such.

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Point (i) implies that the high-energy products of the collision due to the BSW effect can be detected at infinity with extraction of energy from a black hole. However, it was shown recently that in the Kerr background the efficiency of the corresponding process is less than 50 % due to significant red-shift [2], [3]. In more general backgrounds, when a black hole is surrounded by matter, the efficiency can be made larger depending of the details of the metric but in any case some restrictions remain [4]. On the other hand, there exists counterpart of the BSW effect in the Reissner-Nordström (RN) background [5]. It turned out [6] that even for the simplest case of radial motion of test particles, there are scenarios free from restrictions similar to the rotating case. Although the RN metric is not applicable to astrophysical problems directly, it can serve as a useful toy model which shows that high efficiency of the BSW process is possible at least in principle.

Another type of objections concern the effect itself. Here, the role of gravitational radiation remains not quite clear [7], [8]. Another factor that can bound the value of $E_{c.m.}$ is self-gravitation. It was argued in [9] that collision of massive shells do not produce the BSW effect detectable by a remote observer. However, this cannot be considered as objection since in the corresponding situation the light-like surface (which is an essential ingredient of the BSW effect) is hidden for the outer observer. One can think that when instead of thin shells a smeared distribution of matter is present, the BSW effect is restored.

Meanwhile, the objects against the BSW effect have been continuing. Quite recently, a work [10] appeared in which it was stated that for collision of two neutral particles in the "near-horizon geometry" of the extremal Kerr throat the energy in the center–of–mass frame is finite for any value of the particle parameters. (What is called "near-horizon geometry" in [10] actually represents so-called acceleration horizon and hereafter we will use the latter term.) Thus point (ii) mentioned above was touched upon in [10] with the conclusion that, in spite of the presence of the horizon, the BSW effect is absent. Taken literally, the results of [10] would lead to logical contradiction since (a) in the vicinity of an extremal black hole horizon any metric (including the Kerr one) can be approximated by that of the acceleration horizon, (b) it was already shown in [1] and in many other papers that for rotating or charged black holes the BSW effect does exist, (c) by themselves, calculations in [10] seem to be correct. Below we show how to resolve this seeming contradiction.

Apart from this, consideration of the acceleration horizons is a new venue of the BSW effect. Such metrics appear in the process of different limiting transitions in the context of

gravitational thermal ensembles [11] and can be also relevant in the context of the AdS/CFT correspondence [12] as well [13]. All these circumstances can be considered as sufficient motivation for consideration of the BSW effect due to acceleration horizons.

II. FORM OF METRIC

It is well-known that if one takes, say, the Schwarzschild metric and restricts himself by the vicinity of the horizon only, the geometry looks like the direct product of the two-dimensional Rindler metric and the sphere of a constant radius. Further, this approximate geometry can be extended to the whole space, so the configuration $Rindler_2 \times S_2$ can be considered on its own. In doing so, it possesses the acceleration horizon inherited from the two-dimensional Rindler metric. Now, we will see how this approach works for the axially symmetric stationary (but not static) case.

Let us consider the metric

$$ds^2 = -dt^2 N^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + C(\theta, r)\frac{dr^2}{r^2} + g_{\theta\theta} d\theta^2, \tag{1}$$

where the metric coefficients do not depend on t and ϕ. The Kerr metric belongs just to this class. In the present paper, we restrict ourselves by the case when the horizon is extremal. We choose the coordinate r in such a way that $r = 0$ corresponds to the horizon, so for the extremal case $N^2 \sim r^2$ for small r by definition.

Then, the lapse function

$$N = A(\theta, r) r \tag{2}$$

$$\omega = \omega_H + \bar{\omega}, \tag{3}$$

$$\bar{\omega} = -B(r, \theta) r \tag{4}$$

where $A(\theta, r)$ and $B(\theta, r)$ are regular in the vicinity of the horizon, $A(\theta, 0)$, $B(\theta, 0) \neq 0$. The sign "minus" is chosen in (4) here since, say, for the Kerr metric $B(\theta, 0) > 0$. The presentation of the coefficient ω (4) follows from the fact that for regular extremal black holes the first correction to ω_H near the horizon must have the order N [14].

Then, the metric can be rewritten as

$$ds^2 = -dt^2 N^2 + g_{\phi\phi}(d\bar{\phi} - \bar{\omega} dt)^2 + g_{ab} dx^a dx^b, \tag{5}$$

$$\bar{\phi} = \phi - \omega_H t, \tag{6}$$

where $a, b = r, \theta$. The variable $\bar{\phi}$ corresponds to the frame corotating with the horizon. Hereafter, we use the bar sign to denote quantities in this frame. Near the horizon, we may describe the geometry approximately, truncating the metric near the horizon and replacing the coefficients A, B, C and $g_{\theta\theta}$ by their limiting values at $r = 0$. Then, we obtain

$$ds^2 = -A^2(\theta)r^2dt^2 + g_{\phi\phi}(\theta)(d\bar{\phi} + B(\theta)rdt)^2 + C(\theta)\frac{dr^2}{r^2} + g_{\theta\theta}(\theta)d\theta^2. \tag{7}$$

where $A(\theta, 0) \equiv A(\theta)$, etc. The metric (7) belongs to the class (5) with

$$\bar{\omega}_H = 0. \tag{8}$$

If we consider the metric (7) not as near-horizon approximation to (1) or (5) that describe the black hole but, instead, as an exact space-time, we obtain the metric of the acceleration horizon. Alternatively, one can rescale the variables according to

$$r = \varepsilon\tilde{r},\ t = \frac{\tilde{t}}{\varepsilon}, \tag{9}$$

substitute them into (1) or (5) and take the limit $\varepsilon \to 0$. Then, we again obtain (7) with r and t replaced by $\tilde{r}$, $\tilde{t}$.

In a somewhat different form, such a limiting transition was performed in [11], [13]. The resulting geometry is the analogue of the $AdS_2 \times S_2$ one. In particular, nontrivial manifold can be obtained for the vacuum case when the metric (1) describes the Kerr black hole.

III. EQUATIONS OF MOTION

Let a test particle move in the background (1). Then, due to the independence of the metric coefficients of ϕ and t, there are two Killing vectors and two integrals of motion - energy E and angular momentum L. Hereafter, we restrict ourselves by the motion in the equatorial plane $\theta = \frac{\pi}{2}$. Then,

$$m\frac{dt}{d\tau} = \frac{X}{N^2}, \tag{10}$$

$$m\frac{d\phi}{d\tau} = \frac{L}{g} + \frac{\omega X}{N^2}, \tag{11}$$

$$m\frac{\sqrt{C}}{r}\frac{dr}{d\tau} = \varepsilon\frac{Z}{N},\ \varepsilon = \pm 1, \tag{12}$$

$$X=E-\omega L,\ Z=\sqrt{X^{2}-N^{2}(m^{2}+\frac{L^{2}}{g})},\ g\equiv g_{\phi\phi}, \tag{13}$$

where the metric coefficients are taken at $\theta=\frac{\pi}{2}$. Here, m is the particle's mass, τ is the proper time. In (12), we assume the minus sign that corresponds to a particle moving towards the horizon.

We use the system of units where the fundamental constants $G=c=1$.

In a similar manner, for (7) we obtain (10) - (12) with X,Z and ϕ replaced with $\bar{X}$ and $\bar{\phi}$.

For the metric (7), $N=Ar$. The quantity in the original frame and the one corotating with a black hole are related according to

$$\bar{X}=\bar{E}-\bar{\omega}L=X, \tag{14}$$

$$\bar{E}=E-\omega_{H}L\equiv X_{H}, \tag{15}$$

$$L=\bar{L}. \tag{16}$$

$$\bar{Z}=Z=\sqrt{\bar{X}^{2}-N^{2}(m^{2}+\frac{L^{2}}{g})}. \tag{17}$$

A. Classification of particles

As usual, we assume the forward in time condition $\frac{dt}{d\tau}>0$. This entails that $X=\bar{X}\geq 0$. If $X>0$ everywhere we call such a particle usual. Meanwhile, the forward in time condition admits also

$$X_{H}=0 \tag{18}$$

since $N=0$ on the horizon. In such a case, we call the particle critical. Hereafter, subscript "H" means that the corresponding quantity is taken on the horizon. Division of particles into these two classes is crucial for the BSW effect (see below).

For the metric (7), taking into account (8), (18) one obtains that

$$\bar{E}=0 \tag{19}$$

for critical particles and $\bar{E} > 0$ for usual ones. In terms of unbarred quantities the condition of criticality takes a more familiar general form [15]

$$E - \omega_H L = 0. \tag{20}$$

In what follows, we need the asymptotic near-horizon expansions for the quantities X and Z. As near the horizon N is small, we can use the Taylor series. Then, one obtains from (17), (30) that for a usual particle,

$$\bar{X} = \left(\bar{X}\right)_H + BLr + O(r^2), \tag{21}$$

$$Z_i = X_i - \frac{N^2}{2X_i}(m^2 + \frac{L^2}{g_H}) + O(N^3), \tag{22}$$

In a similar way, it follows from (2), (4), (19) (14) that for the critical particle

$$\bar{X} = BLr + O(N^2), \tag{23}$$

$$Z = r\sqrt{B^2L^2 - A^2(m^2 + \frac{L^2}{g_H})} + O(N^2). \tag{24}$$

If such a particle which moves towards the horizon ($\varepsilon = -1$), it follows from (12) that near the horizon,

$$r \approx r_0 \exp(-D\tau), \tag{25}$$

where r_0 is an arbitrary constant and

$$D = \frac{\sqrt{L^2(B^2 - \frac{A^2}{g_H}) - m^2A^2}}{m\sqrt{C}A}, \tag{26}$$

where coefficients A and B are taken at $r = 0$, $\theta = \frac{\pi}{2}$. When $\tau \to \infty$, the critical particle spirals to the horizon asymptotically.

IV. BSW EFFECT

For one particle, the standard textbook formula states that the $E^2 = -p^\mu p_\mu$ where $p^\mu = mu^\mu$ is the four-momentum, $u^\mu = \frac{dx^\mu}{d\tau}$ is the four-velocity. For two colliding particles, in the point of collision one can define the energy in the centre of mass frame as

$$E_{c.m.}^2 = -\left(m_1 u_1^\mu + m_2 u_2^\mu\right)\left(m_1 u_{1\mu} + m_2 u_{2\mu}\right). \tag{27}$$

Then,

$$E_{c.m.}^2 = m_1^2 + m_2^2 + 2m_1 m_2 \gamma \tag{28}$$

where the Lorentz factor of relative motion

$$\gamma = -u_1^\mu u_{2\mu}, \tag{29}$$

$(u^\mu)_i$ is the four-velocity of the i-th particle (i=1,2). Applying this to the metric (7) and using (13), we obtain that

$$\gamma = \frac{1}{m_1 m_2}(c-d),\ c = \frac{X_1 X_2 - Z_1 Z_2,}{N^2},\ d = \frac{L_1 L_2}{g}. \tag{30}$$

Here, we consider different cases separately.

A. Case 1: both particles are usual

It follows from (21), (22), (30) that

$$c_H = \frac{(X_2)_H}{2\,(X_1)_H}(m_1^2 + \frac{L_1^2}{g_H}) + \frac{(X_1)_H}{2\,(X_2)_H}(m_2^2 + \frac{L_2^2}{g_H}). \tag{31}$$

On the horizon, $X_H = \bar{E}$ due to (8) and we obtain from (28), (30) - (31) that

$$E_{c.m.}^2 = m_1^2 + m_2^2 + \frac{\bar{E}_2}{\bar{E}_1}(m_1^2 + \frac{L_1^2}{g_H}) + \frac{\bar{E}_1}{\bar{E}_2}(m_2^2 + \frac{L_2^2}{g_H}) - \frac{2L_1L_2}{g_H} \tag{32}$$

This expression simplifies if we take $\bar{E}_1 = \bar{E}_2$:

$$E_{c.m.}^2 = 2m_1^2 + 2m_2^2 + \frac{(L_1 - L_2)^2}{g_H}. \tag{33}$$

Eq. (33) is valid both for the acceleration horizon described by the metric (7) and the usual black hole metric (1) provided, additionally, $\omega_H = 0$. It is clear from (33) that $E_{c.m.}$ is bounded, so the BSW effect is absent.

B. Case 2: Both particles are critical

Now, according to the definition of critical particles and properties (8), (19), $\bar{E}_1 = \bar{E}_2 = 0$. Then, using (23), (24) one obtains from (28) that

$$E_{c.m.}^2 = m_1^2 + m_2^2 + 2\frac{B_1 B_2 L_1 L_2 - \sqrt{B^2 L_1^2 - A^2(m_1^2 + \frac{L_1^2}{g_H})}\sqrt{B^2 L_2^2 - A^2(m_2^2 + \frac{L_2^2}{g_H})},}{A^2} - \frac{2L_1 L_2}{g_H} \tag{34}$$

The energy is bounded, there is no BSW effect.

C. Case 3: particle 1 is critical, particle 2 is usual

Now, according to (19) $\bar{E}_1 = 0$ but $\bar{E}_2 > 0$. For small N, we have the asymptotic behavior (21), (22) for particle 2 and (23), (24) for particle 1.

Then, by substitution into (30), one obtains that if collision takes place near the horizon, so $N \to 0$,

$$E_{c.m.}^2 \approx 2\frac{(\bar{E}_2)_H}{N}\left(\frac{BL_1}{A} - \sqrt{\frac{B^2}{A^2}L_1^2 - m_1^2 - \frac{L_1^2}{g_H}}\right). \tag{35}$$

Thus in the horizon limit $E_{c.m.}^2$ grows indefinitely, so the BSW effect manifests itself. Eq. (35) has meaning for the angular momenta $L_1^2 \geq m_1^2(\frac{B^2}{A^2} - \frac{1}{g_H})^{-1}$ only. Otherwise, a critical trajectory cannot be realized and the BSW effect is absent.

V. EXAMPLE: VACUUM METRIC

In the simplest case, one can take the extremal Kerr metric as a "seed" one (1):

$$ds^2 = -dt^2(1-\frac{2au}{\rho^2}) - \frac{4a^2u\sin^2\theta}{\rho^2}d\phi dt + \frac{\rho^2}{\Delta}du^2 + \rho^2 d\theta^2 + (u^2+a^2+\frac{2ua^3\sin^2\theta}{\rho^2})\sin^2\theta d\phi^2. \tag{36}$$

Here, u is the Boyer-Linquiste coordinate, $\rho^2 = u^2 + a^2\cos^2\theta$, $\Delta = (u-a)^2$, a characterizes the angular momentum of a black hole. Performing the limiting transition based on the approaches of [11] and [13] described in Sec. II, we arrive at the metric (7) with $r = u - a$,

$$A = \frac{1}{2a}\sqrt{1+\cos^2\theta},\ B = \frac{1}{2a^2},\ C = a^2(1+\cos^2\theta), \tag{37}$$

$$g = \frac{4a^2\sin^2\theta}{1+\cos^2\theta}. \tag{38}$$

For equatorial motion, $\theta = \frac{\pi}{2}$, $A = \frac{1}{2a} = aB$, $C = a^2$, $g = 4a^2$. Then, eq. (35) describing the BSW effect takes the form

$$E_{c.m.}^2 \approx 2\frac{(\bar{E}_2)_H}{N}(\frac{L_1}{a} - \sqrt{\frac{3}{4a^2}L_1^2 - m_1^2}) \tag{39}$$

with $L_1 \geq \frac{2a}{\sqrt{3}}m_1$.

VI. COMPARISON WITH PREVIOUS RESULTS

The formulas under discussion can be also applied to the Schwarzschild case for which $\omega \equiv 0$, $g_H = 4M^2$. Let, for simplicity, $\bar{E}_1 = \bar{E}_2 = m$. Then, a particle falling towards the black hole can reach the horizon provided the constraint $-4M < L < 4M$ is satisfied where M is the black hole mass. Assuming $L_1 = -L_2 = 4M$, one obtains from (33) $E_{c.m.} = m\sqrt{5}$ that reproduces the result of [16]. For arbitrary L_1, L_2 in the Schwarzschild background eq. (33) can be also obtained from eq. (10) of Ref. [17] valid for the Kerr metric if the rotational parameter $a \to 0$.

Eq. (33) agrees also with eq. (17) of [10] if one puts $m_1 = m_2 = m$.

Eq. (35) in the case $m_1 = m_2 = m$ corresponds to eq. (14) of [15] derived for generic axially-symmetric black holes.

VII. DISCUSSION AND CONCLUSIONS

The results obtained in the present paper apply to two different objects: (i) they can be understood as describing the BSW effect near rotating black holes in the frame corotating with the black hole horizon, (ii) they also apply to acceleration horizons when a black hole as such is absent. By itself, point (i) is not very interesting since it reproduces the results already obtained earlier. However, as the vicinity of the black hole is approximately described by the metric of the acceleration horizon, point (i) in combination with (ii) resolves the seeming contradiction mentioned in Introduction and suggests an unified framework for the description of the BSW effect for black holes and acceleration horizons.

The main new result is contained in point (ii). We showed that, contrary to the claims made in the recent work [10], the BSW effect does exist for rotating acceleration horizon like the Extremal Kerr throat [13], so it cannot serve as a regulator which would allow to avoid the BSW effect. It is worth stressing that, although the metric of the form (7) can be obtained from the black hole one (1), its properties can be quite different. In particular, the flat infinity can be absent (see [13] for detailed discussion of the geometry obtained from the Kerr throat).

The reason of discrepancy between the main claims of [10] and of our work is quite simple. It was assumed in [10] that energies of the colliding particles $E_1, E_2 > 0$. However, these

particles, in our terminology, are *usual*. It was shown earlier [18] in a more general context, that collisions between two usual or two critical particles cannot produce the BSW effect. The results of [10] refer to what is denoted above as case 1. Meanwhile, there is also case 3 which is the most interesting: one of particle is critical, the other one is usual. In accordance with the general scheme it leads to the BSW effect. In doing so, the critical condition (19) reads $\bar{E}_1 = 0$ that does not fall into the class of collisions considered in [10], so the BSW effect was overlooked there.

Although, on the first glance, condition (19) looks unusual, it is equivalent to the standard condition of criticality (20) relevant for BSW effect near black holes. Moreover, the corresponding condition (18) is the same for the original and rotating frames since X is invariant under transformation from one frame to another, so (19) and (20) are different manifestation of the same property. Therefore, there is continuity between two kinds of the effect - due to black hole and acceleration horizons.

The geometries like (7) is the counterpart of the $AdS_2 \times S_2$ one typical of the spherically symmetric case. Bearing in mind their possible role [13] in the context of the AdS/CFT correspondence in quantum field theory, one can think that the relevance of the BSW effect in such geometries can be of potential interest not only in astrophysics but in particle physics as well.

The results of the present paper are in agreement with the fact that, actually, the BSW effect is based on two ingredients: (i) the existence of the horizon, (ii) the presence of a critical particle. Then, the BSW should take place even for nongeodesic motion of neutral particles [19].

One reservation is in order. We neglected backreaction and gravitational radiation which can influence the general picture near the horizon [8], [7]. Meanwhile, the aforementioned results of [19] lend support to the idea that these factors can be compatible with the BSW effect. This issue is far from being clear and needs separate treatment.

Note added. Quite recently, a new version of preprint [20] appeared in which its author states: "The situation considered in [16] (this preprint - O.Z) implies the vanishing energy for one of the colliding particles, which is apparently meaningless in the relativistic context." This general statement is incorrect. It is just strong gravitational field (i.e. relativistic context) that makes it possible to have not only zero but even negative energy. For example, this happens in the ergoregion of the Kerr metric as is explained in many textbooks. The

negative energy is the key ingredient in the the celebrated Penrose process and superradiance. My statement about logical contradiction between [10] and [1] is left in [20] without reply[1].

Acknowledgments

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